The $\overline{\partial}$-Neumann problem and Schrödinger operators

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We apply methods from complex analysis, in particular the $\overline{\partial}$-Neumann operator, to investigate spectral properties of Schrödinger operators with magnetic field (Pauli operators). For this purpose we consider the weighted $\overline{\partial}$-complex on $\mathbb{C}^n$ with a plurisubharmonic weight function φ. Let $1 \leq q \leq n-1$.

$$L^2_{(0,q-1)}(\mathbb{C}^n, e^{-\varphi}) \underset{\overline{\partial}^*_\varphi}{\overset{\overline{\partial}}{\rightleftarrows}} L^2_{(0,q)}(\mathbb{C}^n, e^{-\varphi}) \underset{\overline{\partial}^*_\varphi}{\overset{\overline{\partial}}{\rightleftarrows}} L^2_{(0,q+1)}(\mathbb{C}^n, e^{-\varphi})$$

and

$$\Box_{\varphi,q} = \overline{\partial}\, \overline{\partial}^*_\varphi + \overline{\partial}^*_\varphi \overline{\partial}.$$

We derive a necessary condition for compactness of the corresponding $\overline{\partial}$-Neumann operator (the inverse of $\Box_{\varphi,q}$) and a sufficient condition, both are not sharp. So far, a characterization can only be given in the complex 1-dimensional case.
The Pauli operators appear at the beginning and at the end of the weighted $\overline{\partial}$-complex.
It is also of importance to know whether a related Bergman space of entire functions

$$A^2(\mathbb{C}^n, e^{-\varphi}) = \{f : \mathbb{C}^n \longrightarrow \mathbb{C}\,\text{entire} : \int_{\mathbb{C}^n} |f|^2 e^{-\varphi} d\lambda < \infty\}$$

is of infinite dimension. The main results are formulated in terms of properties of the Levi matrix

$$M_\varphi = \left(\frac{\partial^2 \varphi}{\partial z_j \partial \overline{z}_k}\right)_{j,k=1}^n$$

of the weight function. If the weight function is decoupled

$$\varphi(z_1, \dots, z_n) = \varphi_1(z_1) + \dots + \varphi_n(z_n),$$

one gets additional informations.
Finally we point out that a corresponding Dirac operator fails to be with compact resolvent.

References

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